

MATH 2230 Complex Variables with Applications
(2014-2015, Term 1)
Suggested Solution to HW3

1. Prove that any rational function can be written as a sum of partial fractions.

Proof: Refer to lecture notes 6.

2. As an example write the following function into sum of partial fractions:

$$\frac{z^4 + z^3 - 2z^2 + 1}{z^3 + 2z^2 + z}$$

Solution:

$$\frac{z^4 + z^3 - 2z^2 + 1}{z^3 + 2z^2 + z} = z - 1 + \frac{-z^2 + z + 1}{z^3 + 2z^2 + z}$$

Note that the roots of the polynomial $z^3 + 2z^2 + z$ are 0, -1 with order 1 and 2 respectively.

Then we let

$$\frac{-z^2 + z + 1}{z^3 + 2z^2 + z} = \frac{A}{z} + \frac{B}{z + 1} + \frac{C}{(z + 1)^2}$$

Thus, $A = 1, B = -2, C = 1$.

Therefore,

$$\frac{z^4 + z^3 - 2z^2 + 1}{z^3 + 2z^2 + z} = z - 1 + \frac{1}{z} - \frac{2}{z + 1} + \frac{1}{(z + 1)^2}$$